

v/vmp Beats and beat frequency \Rightarrow (1)

amplitude but slightly different frequency move in the same direction then the waves superimpose to each other giving rise to alternating hearing of the sound. This phenomenon is called beats.

The ~~total~~ number of beats formed per second is called beat frequency.

Expression of beat frequency \Rightarrow

Suppose, two waves of frequencies f_1 and f_2 travelling in the same direction superimpose on each other.

The equations of two waves are given by,

$$y_1 = a \sin \omega_1 t \quad \text{--- (1)}$$

$$y_2 = a \sin \omega_2 t \quad \text{--- (2)}$$

where, a is amplitude.

since, $\omega = 2\pi f$; ~~where ω is of frequency~~

$$\text{Then, } y_1 = a \sin 2\pi f_1 t \quad \text{--- (3)}$$

$$\& y_2 = a \sin 2\pi f_2 t \quad \text{--- (4)}$$

so, the resultant displacement is,

$$y = y_1 + y_2$$

$$\begin{aligned} \therefore y &= a \sin 2\pi f_1 t + a \sin 2\pi f_2 t \\ &= a [\sin 2\pi f_1 t + \sin 2\pi f_2 t] \end{aligned}$$

$$y = a \cdot 2 \sin\left(\frac{2\pi f_1 t + 2\pi f_2 t}{2}\right) \cdot \cos\left(\frac{2\pi f_1 t - 2\pi f_2 t}{2}\right) \quad (2)$$

$$\left[\because \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \right]$$

$$\therefore y = 2a \cdot \cos\left[\frac{2\pi \cdot (f_1 - f_2) \cdot t}{2}\right] \cdot \sin\left[\frac{2\pi \cdot (f_1 + f_2) t}{2}\right]$$

$$\therefore y = A \sin 2\pi \left(\frac{f_1 + f_2}{2}\right) \cdot t \quad \text{--- (5)}$$

where, $A = 2a \cos\left[\frac{2\pi (f_1 - f_2) t}{2}\right]$ is the resultant amplitude

• Now, the amplitude (A) will be maximum,

$$\text{if } \cos\left[\frac{2\pi (f_1 - f_2) t}{2}\right] = \pm 1$$

$$\text{or, } \cos\left[\frac{2\pi (f_1 - f_2) t}{2}\right] = \cos n\pi \quad ; \text{ where, } n = 0, 1, 2, \dots$$

$$\therefore \frac{2\pi (f_1 - f_2) t}{2} = n\pi$$

$$\therefore (f_1 - f_2) t = n$$

$$\therefore t = \frac{n}{f_1 - f_2} \quad \text{--- (6)}$$

putting values of n then from eq. (6), we get,

$$\therefore t = 0, \frac{1}{f_1 - f_2}, \frac{2}{f_1 - f_2}, \dots$$

$$\text{i.e. Time period (T) = } \frac{1}{f_1 - f_2}$$

$$\therefore \text{beat frequency (f) = } \frac{1}{T} = f_1 - f_2 \quad //$$